Math 456: Topology and Its Applications
Homework 8

Due Monday, November 6th

1. Consider the following filtration of simplicial complexes with the canonical inclusion maps.

$$S_1 = ([4], \{12, 14, 23, 34\}) \hookrightarrow S_2 = ([4], \{12, 14, 23, 34, 24\}) \hookrightarrow S_3 = ([4], \{12, 14, 234\})$$

   (a) Write down a basis for $H_1(S_1)$ and $H_1(S_3)$, and *three* different bases for $H_1(S_2)$.

   (b) Using these three choices of basis, write down matrices for the induced maps $(\iota_{1,2})_1$ and $(\iota_{2,3})_1$ on H_1. Check that the composition is the same no matter which choice of basis you make.

   (c) Discuss the complexity that arises in describing "which" cycle from S_2 becomes the generator in S_3.

2. Let $D_1 = \{(1,2), (1,4), (2,3), (3,4)\}$ and $D_2 = \{(1,3), (2,3), (2,4)\}$ be persistence diagrams.

   (a) Compute the bottleneck distance $d_B(D_1, D_2)$.

   (b) Draw the persistence landscapes $\Lambda(D_1)$ and $\Lambda(D_2)$.

   (c) Draw the mean persistence landscape $\frac{1}{2}(\Lambda(D_1) + \Lambda(D_2))$.

3. Write a Julia function which samples points uniformly at random from the sphere S^2 by selecting an ordered pair of points from the normal distribution and them normalizing them to length 1.

   (a) Use this function to sample a list of 50 points and have Eirene compute the persistent homology in dimensions 0, 1, and 2 for the sample. Plot the persistence diagrams, barcodes and Betti curves in each dimension. Plot the generator for the long-lifetime class in dimension 2.

   (b) Take 50 samples each of 30 points, 60 points and 100 points from the sphere, and compute persistent homology in dimensions 0, 1 and 2 for each sample. On the same axis, plot the average Betti curves for each set of samples. What do you observe?