Math 456: Topology and Its Applications
Homework 7

Due Friday, October 27th

1. Let $P \subseteq \mathbb{R}^3$ be a point cloud uniformly and densely sampled from the surface of a torus given by

   $$x(\phi,\theta) = (1 + 0.5\cos(\phi))\cos(\theta) \qquad y(\phi,\theta) = (1 + 0.5\sin(\phi))\cos(\theta) \qquad z(\phi,\theta) = \sin(\phi)$$

   Draw the result of the mapper algorithm for

   (a) $(x, y, z) \mapsto x$ with $\mathcal{U} = \{(-\infty, -1/3), (-1/2, 1/2), (1/3, \infty)\}$
   (b) $x, y, z) \mapsto z$ where $\mathcal{U} = \{(-\infty, -1/4), (-1/3, 1/3), (1/4, \infty)\}$

   Can you find a circle in $\mathbb{R}^3$ which gives the same output for these two computations? If so, how might you go about differentiating the two objects using mapper? If not, why not?

2. One possible set of vertices of a regular dodecahedron are given by

   $$(\pm 1, \pm 1, \pm 1) \qquad (0, \pm\phi, \pm\frac{1}{\phi}) \qquad (\pm\frac{1}{\phi}, 0, \pm\phi) \qquad (\pm\phi, \pm\frac{1}{\phi}, 0)$$

   where $\phi = (1 + \sqrt{5})/2$. Draw the barcode for the persistent homology of the Vietoris-Rips complex and the Čech complex for this set of points.

   Don't go all the way to the definition of homology and compute (unless you really want to); rather, use geometry and symmetry to make an argument about what the homotopy type of the simplicial complex will be for various ϵ and use what we know about those homotopy types to work out the persistent homology. Be exact with the distances, don't use decimal approximations.

3. Let

   $$M = \begin{pmatrix} 0.3 & 0.9 & 0.7 \\ 0.2 & 0.3 & 1.1 \\ 0.4 & 0.5 & 0.3 \\ 0.8 & 0.1 & 0.6 \\ 0.9 & 0.3 & 0.2 \\ 1.1 & 0.7 & 0.2 \end{pmatrix}.$$

   Compute $H_*(\mathrm{Dow}_{0.5}(M))$.

4. Prove the following lemma from the lecture notes.

   Lemma. *Let $P \subset \mathbb{R}^d$ be a point cloud. Then $\check{C}_\epsilon(P) \subseteq VR_\epsilon(P) \subseteq \check{C}_{2\epsilon}(P)$.*

5. On your favorite computer, install Julia (julialang.org) and Eirene (gregoryhenselman.org). Get Eirene running; when it's working properly, executing the command INCLUDE("EIRENE0_3_7.JL") will execute without a sequence of lines beginning "PLEASE

NOTE:", and the output will end with EIRENE (GENERIC FUNCTION WITH 2 METHODS). Play around a bit and get comfortable with Julia – there are several good tutorials in the "Learning" section of the Julia web page. Use Eirene to check your work on problem 2. (This will require approximating ϕ – c'est la vie.) You don't need to hand this in.